OR/MS Games: 1. A Neglected Educational Resource

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Abstract

Games offer OR/MS lecturers and students important educational resources. However, for various reasons the OR/MS literature in general - and introductory textbooks in particular - scarcely make use of this treasure. In this discussion we suggest that it is time to reconsider this position and regard games as a valuable source for developing educationally rich material for OR/MS courses. Hopefully this will initiate a constructive discussion on this topic that will eventually facilitate the identification and development of good OR/MS educational resources and stimulate the development of new exciting ways to make OR/MS tools more accessible to students and the public at large.

1. Introduction

Games, puzzles, and paradoxes have long been recognised as important tools of thought in education. They can be used in a variety of ways including:

- Alternative formats for conventional lectures.
- Content material for courseware.

For example, Cochran (2001) recently reported on the use of the format of the popular TV game show *Who Wants to be a Millionaire®* to alter the format of short parts of conventional lectures so as to deal with student attention level and Rump (2001) showed how dynamic programming can be used to solve this kind of games. Trick (2001) vividly describes the use of a game (Flip) as content material for an MBA course. Chlond and Toase (2002) mention socio-cultural reasons for the incorporation of games in OR/MS courseware.

Our discussion is dedicated solely and exclusively to the second item listed above, namely to the use of games as content material for OR/MS courseware.

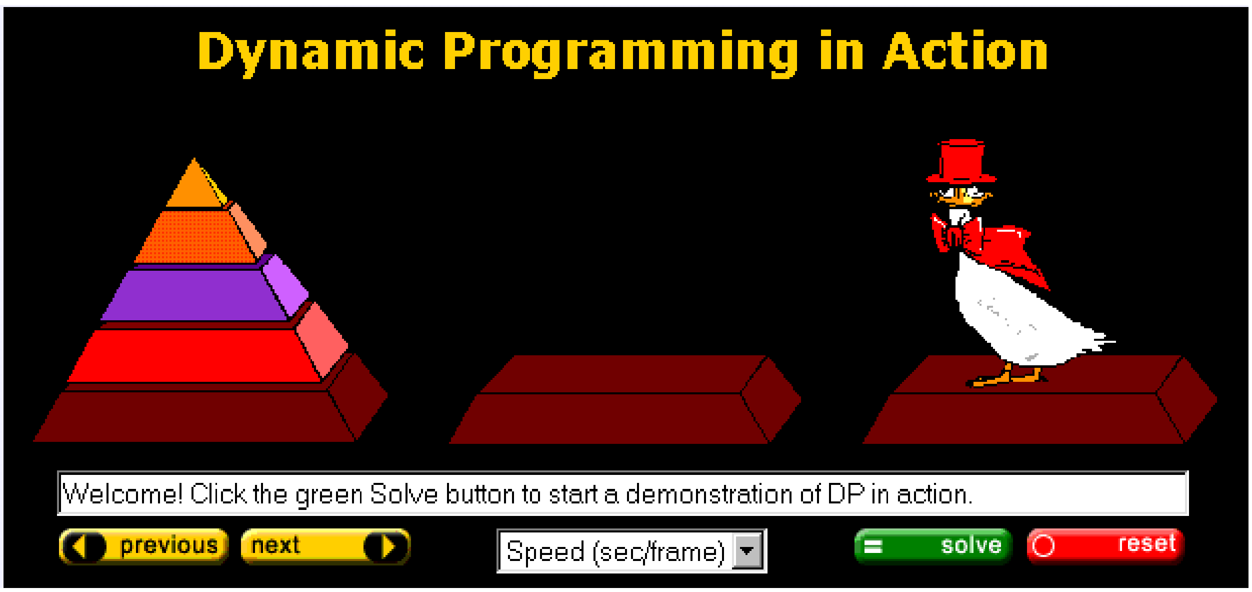
A quick search of Yahoo's directory of Math Problems, Puzzles and Games reveals that games are important ingredients in math education. The following quote tells part of the story:

> *Although I cannot define a mathematical game any better than I can a poem, I do maintain that, whatever it is, it is the best way to capture the interest of young people in teaching elementary mathematics. A good mathematical puzzle, paradox, or magic trick can stimulate a child's imagination much faster than a practical application (especially if the application is remote from the child's experience), and if the "game" is chosen carefully it can lead almost effortlessly into significant mathematical ideas.*
>
> Gardner (1990, p. xi)

Given the extremely important role that mathematics play in many areas of OR/MS it is surprising that games, puzzles and paradoxes have not been used extensively in OR/MS education. Apparently one reason why some OR/MS lecturers are reluctant to incorporate games in their courseware is the notion that games are usually not realistic enough for OR/MS students (see for example comments made by Trick (2001). That is, games usually do not do a good job in representing 'real life' problems. Thus, it is difficult to justify their inclusion in lectures, labs, tutorials, quizzes and exams in courses whose stated goal is to train students in solving practical problems that they are likely to encounter on the job.

The author's view is that games can play an important role in OR/MS education. They should, of course, be used in moderation and be integrated in a suitable manner with other ingredients of the courseware. In this regard they are like spices used in preparation of food.

The following very old implementation of the famous Towers of Hanoi game should set the tone for our discussion. (You need a browser supporting JavaScript to be able to interact with this module, eg. recent versions of Netscape and MS Internet Explorer).



As we shall see in a subsequent article, this old and very popular puzzle is very rich educationally. By this we mean that it provides a framework for the analysis of a number of important problem-solving concepts that are commonly used in such disciplines as Applied Mathematics, Operations Research, Management Science, Computer Science, Artificial Intelligence, Constraint Programming and so on. Given that many of its modelling and computational aspects lend themselves to elegant dynamic programming treatments, it is a bit surprising that it is not used much in introductory OR/MS textbooks. Furthermore, it is a pity that dynamic programming is often not credited for the popular recursive solution method for this problem (eg. Brassard and Bratley, 1988, p. 64).

The main objective of this paper is to initiate a constructive discussion on the potential use of games in OR/MS Education. We argue that it is time for OR/MS lecturers to reconsider their position with regard to the use of games as important ingredients of their total OR/MS courseware. We announce the creation of a depository of OR/MS games designed specifically for teaching/learning purposes.

In subsequent articles on this and related topics we shall examine some of the games mentioned in this article in detail and analyse their educational content in the context of OR/MS courses. In anticipation of this, we provide (Section 7) a short mathematical exposition of dynamic programming's approach to solving a variety of games.

2. What is an OR/MS game?

The notion of "game" - whatever it really is - is strongly related to that of "play", which to many is the opposite of "work". Of course Person A's play can be Person B's work and vice versa. And there are those lucky ones for whom "work" and "play" are one and the same thing.

In view of this it would be best to leave the notion 'game' undefined in this discussion. Intuitively it can be regarded as a problem or task of a recreational rather than work-related nature. Thus, in this spirit an OR/MS game is a problem or task with strong

OR/MS flavour that is recreational rather than work-related.

According to this rather (purposely) vague description of what an OR/MS game is, it is possible that to some OR/MS students the Travelling Salesman Problem and Knapsack Problem would look, smell and taste like games par excellence, whereas to others the N-queen Puzzle will not look like a game at all.

Indeed, one particular puzzle, namely the '14-15' puzzle, invented by Sam Lloyd (1841-1911) became an international hit and attracted a lot of attention because of the pretty large prize ($1000) offered to the first correct solution. The prize was never collected mainly because Lloyd made sure that the instance of the puzzle used was insoluble. This puzzle apparently had a major positive side effect: it indirectly revived interest in Fermat's Last theorem! (See Singh 1998, pp. 142-143)!

In any case, in view of the above it should be clear that OR/MS games are everywhere. This is so because the area of combinatorial optimisation is right at the heart of OR/MS and many games have to do with combinatorial optimisation. Differently put, many games can be described - in a natural manner - as combinatorial optimisation problems, hence as legitimate OR/MS problems.

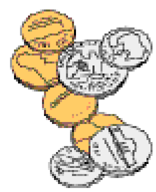
Last but not least, it should be stressed - just in case - that the notion 'game' as is used here should not be confused with problems falling under the jurisdiction of Game Theory. It should be pointed out though that some of the founders of Game Theory, especially John Nash, definitely had very strong interest in recreational games (see Nasar 1998).

3. Examples

In this section six games are briefly described. These game have been chosen for this purpose because they have been used by the author extensively over the past twelve years in his OR/MS related teaching. They have been implemented on the Web and are freely accessible via the tutOR and tutORial web sites.

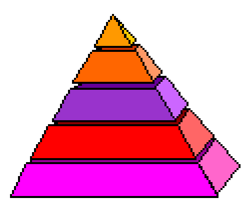
The best way to learn about these games is to ..... play them. The short descriptions given below are intended to give a short OR/MS perspective on these games rather than to explain in detail what they are and how they are played.

Counterfeit Coin Problem



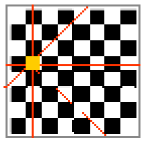
You are given a collection of N coins all of which have the same weight except one which is (slightly) heavier than the others. You are also given a balance beam. The objective is to determine a weighing scheme to identify the counterfeit (heavy) coin as fast as possible. This basic problem has several versions, depending on how Mother Nature decides where to hide the false coin, number of false coins and their properties. It is a good framework for introducing basic concepts associated with decision making under uncertainty, including the minimax and pigeon hole principles. Its stochastic (expected value and probabilistic) versions are good frameworks for the introduction of stochastic dynamic programming techniques.

Towers of Hanoi Problem



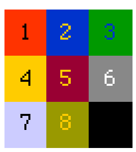
N pieces of different size are placed on a platform so that each piece is on top of a larger piece. The task is to move these pieces to another platform - one at a time - and arranged them there in the same order. A third platform is provided for temporary placement of the pieces. The arrangements of pieces on this platform is subject to the same restriction as the other two platforms. Your task is to determine a scheme for accomplishing this task with the minimum number of moves. This game is very popular in the computer science literature where it is used for several purposes, among them to illustrate recursive algorithms in action. There are extensions to this game such as games where there are more than 3 platforms (so called multipeg games).

N-Queen Problem



You are given an NxN checker board. Your mission is to place as many queens as possible on the board so that none of them attack any of the others. This is one of the most popular examples used in introductory expositions of constraint programming and related topics. It is a very tempting playing grounds for heuristics methods and illustrating the notion of backtracking and symmetries. From a modelling point of view it is a very nice relative of the classical assignment problem. It is a pity that it is not used more extensively in introductory OR/MS text books. An integer programming formulation of this game is discussed by Chlond and Toase (2002.)

Eight Easy Pieces



You are given an 3x3 board with 9 distinct tiles on it, one of them is 'blank'. The tiles are then shuffled. Your mission is to restore the tiles to their original positions in a minimum number of moves. A move consists of interchanging the blank tile with one of its neighbours. There are other variations on this theme, including 4x4 games and games where the initial arrangement of the tiles represents a picture so the game is all about restoring the original picture. It is a natural environment for the Manhattan distance. It is a great framework for an introductory lecture on branch and bound. Sam Lloyd's famous '14-15' puzzle, is an instance of the 4x4 set-up, where the positions of tile 14 and tile 15 are interchanged. This instance of the puzzle is insoluble.

Prince's Pub Problem

You are given one 8 oz mug, one 5 oz mug and one 3 oz mug. Initially the 8 oz mug is full, the others are empty. Your mission is to transfer half of the content of the 8 oz mug to the 5 oz mug through a sequence of pouring. The mugs are not marked so unless a mug is full or empty it is not possible to determine the volume of liquid it contains by simple inspection. This puzzle featured in one of the Die Hard movies where Bruce Willis (who else?!) must solve such a game to prevent a major disaster. Given this very important application of the generic problem represented by the game, it would be better perhaps to upgrade the status of the generic problem and not regard it as a game any more.

Gee Park Problem

Given the location and height of a hurdle and the value of the gravitational constant on a given planet, determine the vertical and horizontal velocities of a moving body on that planet, as well as the take-off position, so as to just clear the hurdle. This simple animation is very effective in dealing with some counter intuitive features of the motion of free bodies subject to gravitational forces and the impact of the gravitational constant on these features. It can be used to illustrate the role of nonlinear optimization in animation.

The first two puzzles will be discussed in detail in subsequent articles.

In the next section we briefly discuss the dilemma that some OR/MS courseware developers face regarding the use of such games in their classrooms.

4. The Dilemma

To explain the dilemma faced by OR/MS lecturers who are inclined to incorporate games in their courseware let us consider the case of 'Mathematical Modelling'. So suppose that your next two lectures are dedicated to 'The Art of Mathematical Modelling and its Role in OR/MS'. Your plan is to teach the students how to translate wordy descriptions of real-life problems into mathematical models of the type used in OR/MS. The basic question is this: Should you or shouldn't you incorporate games in your next two lectures?

This is not an easy question.

The crux of the matter is this: if the stated goal of your course is to teach students how to deal with practical OR/MS problems then surely the OR/MS content of the concrete examples you use in your lectures cannot be ignored. Ideally then, the examples you use should have a considerable OR/MS flavour and be as 'realistic' as possible.

But as we know only too well, realistic OR/MS problems tend to be complex and are usually cluttered with little digressions. They can be excellent material for some subjects but are not particularly good teaching/learning material for introductory expositions of OR/MS subjects dealing say with generic methods, algorithms, and heuristics.

It has been the author's experience that the key to success in this area is the maintenance of a proper balance: Games can be extremely effective when used sparingly and applied at strategic places of an exposition of a complicated subject. Excessive use of games may dilute the perceived OR/MS content of the exposition below a critical level.

The optimal game dosage for an OR/MS courseware may vary significantly depending on the background of the students and their expectations from the course. OR/MS students who are 'mathematically mature' as well as 'practically oriented' tend to benefit from larges dosages of math games as they are capable of easily identifying the relevance of the mathematical content of a game to practical OR/MS problems.

The OR/MS topic under consideration is also an important factor in determining the desirable level of game dosage. The author tend to use a very low game dosage in LP oriented subjects and a much larger dosage in DP and Mathematical Modelling oriented subjects.

There are circumstances where the dilemma is resolved in a straight forward manner. In particular, if OR/MS methods are called upon to assist in the design and marketing of a new game then the game constitutes a realistic real life OR/MS problem. Trick (2001) has recently discussed such a case.

5. Potential Role of Games in the OR/MS Classroom

Games can be used effectively in most of the topics related to the traditional problem-solving process (formulation -> modelling -> analysis -> solution). For the purposes of this discussion it is worthwhile to briefly mention the following areas:

Mathematical Modelling

Many students find it difficult to develop mathematical models for given "wordy" descriptions of problems. The above games complement very nicely standard OR/MS problems and can be used in a variety of ways to train students in the art of mathematical modelling. For example, the N-Queen problem is a variate of the classical Assignment Problem, and the 8 Easy Pieces Puzzle and the Towers of Hanoi Problem can be viewed as typical sequencing/scheduling problems.

Simulation

Simulation models are important tools in the analysis of the behaviour of complex systems. They are invaluable in explaining counter intuitive aspects of involved problems. For example, a simple animation of the Gee Park Problem provides a very effective way of explaining common counter-intuitive behaviours of bodies in free motion. A simulation model of the Counterfeit Coin Problem provides a very vivid explanation of the difference between the minimax and expected value decision criteria used in decision making under uncertainty. It also provides an excellent illustration of the Pigeon Hole Principle.

Algorithms

Animation can be very useful in explaining the working of OR/MS algorithms and in testing students' understanding of the various aspects of these procedures. For example, the Counterfeit Coin Problem and the Towers of Hanoi Problem provide excellent frameworks for experimentation with simple dynamic programming algorithms.

Heuristics

Simulation models are useful tools for testing the performance of heuristics. Games of the type listed above lend themselves to simulation through simple animation and thus enable students to experiment with and test the performance of heuristics. They are also useful in explaining why sometime greedy heuristics yield optimal policies. The Counterfeit Coin Problem is a case in point.

Pattern Recognition and Analysis

By analysing the behaviour of a simulation model of a game it is often possible to recognise patterns and use them constructively in the analysis and solution of the game. Major tools of analysis, such as 'proof by induction' rely on detection and quantification of patterns. The N-Queen Problem and the Towers of Hanoi Problem provide excellent environments for illustrating pattern recognition and analysis in action.

OR/MS Software

It is important for OR/MS students to be aware of the capabilities and limitations of 'standard' OR/MS software packages. Games provide a rich source for problems on which students can experiment. It is a pity that commercial OR/MS software developers do not use games more extensively in tutorials they provide for their products.

Within the framework of the total courseware package provided for an OR/MS course, games can be used in lectures, assignments, tutorials, lab sessions, projects, quizzes and exams. They are particularly suitable for self-study and 'supplementary reading'.

Although it is not absolutely necessary to actually play with a 'real' implementation of a game to make use of its pedagogical features - a 'pencil and paper' version may suffice - an interactive version of a game can make it educationally richer. Needless to say, the web is a natural delivery system for such implementations.

In the next section we briefly mention three inter-related web-based projects featuring - in addition to conventional OR/MS problems and methods - also games.

6. tutOR, tutORial and IOE Projects

tutOR is a project dedicated to the development of web-based tutorial modules for OR/MS topics. It features over twenty interactive modules covering linear algebra, linear programming, integer programming and dynamic programming topics. At present all the modules are freely accessible via its web site. Some of these modules also feature in the tutORial project. This is an initiative of the International Federation of Operational Research Societies (IFORS) that was inspired by the tutOR project.

These two projects recognise the important role that games can play in OR/MS education and are actively involved in the development of web based interactive OR/MS tutorial modules based on games. In fact, all the modules incorporated in this article are tutOR modules. Due to copyright and intellectual property considerations, some games have two version: a "poor man's version" and a jazzed-up version. The former are used in tutORial the latter in tutOR.

Modules of the tutORial project are incorporated in the IFORS On-line Encyclopedia (IOE). It is envisaged therefore that IOE will eventually feature a substantial number of interactive OR/MS games. Both projects will be officially launched at the IFORS 2002 Conference, but their web sites are already open for public preview.

The game modules in tutOR, tutORial and IOE are purposely designed to be used in a stand-alone, self-contained mode. They can thus be used in conjunction with other web pages where supplementary material is provided to suit the particular needs of the user. This way, the same basic module can be used for a number of purposes, eg. introductory and advanced level treatments of a given topic.

7. Math detour

From a mathematical point of view many games can be formulated as sequential decision problems. They can be captured by the following generic math model:

$$f(s) = \min_{x_1,\ldots,x_k} c(s_1,x_1) + \ldots + c(s_k,x_k) \qquad (1)$$

$$\text{s.t.} \quad s_1 = s \qquad (2)$$

$$s_{k+1} = s \qquad (3)$$

$$s_{j+1} = T(s_j,x_j)\ ,\ j = 1,2,\ldots,k \qquad (4)$$

$$x_j \text{ in } D(s_j)\ ,\ j = 1,2,\ldots,k \qquad (5)$$

x_j = decision made at stage j of the process.

s_j = state of the process at stage j.

$c(x_j,s_j)$ = cost generated by s_j and x_j.

s = current state of the process (used as a parameter).

s = terminal state of the process (given).

$D(s_j)$ = set of feasible decisions when the process is in state s_j.

T = Transition function. That is, $s_{j+1} = T(s_j,x_j)$ is the next state of the process given that the current state is sj and the current decision is x_j.

where

In words, the objective is to minimize the total cost generated when the process is transformed from the current state to a given terminal state subject to the transition function of the process (T) and the feasibility conditions (imposed by D).

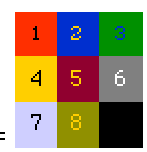
In many instances the cost function is degenerate, that is c(s,x)=1 for any feasible (state,decision) pair. In such cases the goal is to minimize the number of transitions required to transform the process from the given initial state to the given terminal state. Under this condition (1) takes the following degenerate form:

$$f(s) = \min_{x_1,\ldots,x_k} k \qquad (6)$$

As an example, consider the Eight Easy Pieces game. We can let the state variable be a matrix representing a feasible arrangement of tiles, so that s is the current arrangement of tiles and σ is the desired arrangement. Also, let the decision variable be the tile we move (interchange with the blank tile). Thus, $D(s_j)$ is the set of all the tiles that are neighbours of the blank tile in the arrangement stipulated by matrix s_j, and $T(s_j,x_j)$ is the new arrangement of tiles if given the arrangement s_j we interchange tile x_j with the blank tile.

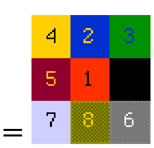
For instance, consider the case where

s = 

, σ **=** 

Then D(s)={2,3}, D(s)={6,8}, and

T(s,2) = 

, T(s,3) = 

It is easy to modify the above formulation to deal with the 4x4 version of the problem. In fact, it is almost as easy to modify the above formulation to deal with the (general) NxN problem.

Exercise: Construct a sequential decision model for the Towers of Hanoi problem and the Prince's Pub problem.

The dynamic programming functional equation for the model (1)-(5) is as follows:

$$f(s) = \min d \text{ in } D(s) \ \{c(s,d) + f(T(s,d))\} \ , \ s \text{ in } S \quad (7)$$

where S is the state space (consisting of all the states other then the terminal state s). For the terminal state we have (by definition) $f(\sigma)=0$.

In the case of (6), the functional equation takes the following form:

$$f(s) = 1 + \min d \text{ in } D(s) \ f(T(s,d)) \ , \ s \text{ in } S. \quad (8)$$

These functional equations can be solved in a variety of ways depending on the structure and properties of the sequential decision model (Sniedovich, 1992).

Of course, this is not a fool proof recipe. In particular, the dynamic programming functional equations of some games are subjected to the Curse of Dimensionality: the number of states increases dramatically (exponentially or even worse) with the size of the problem. Thus, an important aspect of solving OR/MS games is dealing with the Curse of Dimensionlity when necessary. Needless to say, such games provide excellent frameworks for discussions on the complexity of problems and algorithms and the difference between these two important notions of complexity.

In addition, in some games (eg. Eight Easy Pieces) it is necessary to avoid cycles and this may further aggravate an already difficult task.

However, these difficulties should not be regarded as obstacles, but rather as a challenge. In particular, they present plenty of opportunities for the application of OR/MS methods and techniques and thus make these games educationally rich from an OR/MS perspective.

8. Math-free zone

Many OR/MS classes are populated by students whose mathematical background is limited. In this regard OR/MS games are not different from mainstream OR/MS topics such as say linear programming, dynamic programming, duality theory, queueing theory, and inventory control - to mention just a few. Our libraries are full with textbooks that provide (almost) math-free expositions of these topics.

This does not mean, however, that such students cannot cope with results obtained by hard-core mathematical treatments of OR/MS games. What this does mean is that lecturers must pay attention to this issue and develop the courseware accordingly.

The message is then that OR/MS games are amenable to a wide range of expositions and analyses with regard to the sophistication of the mathematical tools and concepts used, including the math-free variety. The challenge is then not in developing methods for subjecting math-phobic students to a deep math analysis of OR/MS games, but rather to develop math-free expositions of such games for math-free OR/MS oriented courses and programs.

For example, the counterfeit coin problem provides an educational rich environment for a discussion on decision making under uncertainty and can be incorporated in math-free treatments of this topic. In particular, it can be used effectively in a math-free treatment of decision-trees.

9. What's next?

OR/MS methods are well suited for the analysis and solution of 'practical' games. As illustrated by Trick (2001), OR/MS methods such as dynamic programming can be used in the design, analysis and marketing of new games. Given the very strong combinatorial nature of many games, it seems that GAMES could become an important and challenging application area of OR/MS. In particular, Chlond and Toase (2002) demonstrated that integer programming formulations can be developed for many chessboard placements and related puzzles.

Should this happen, the dilemma discussed in this paper will be resolved in a most satisfactory manner. Until then OR/MS lecturers will have to deal with this dilemma more or less in an ad hoc manner and therefore it will be useful to discuss this matter in OR/MS forums. This journal offers an excellent framework for this purpose.

A small step in this direction was taken already by the tutOR and tutORial projects, which now have directories dedicated to OR/MS games. The IFORS On Line Encyclopdia will also have an entry dedicated to OR/MS games.

Also, Chlond's Integer Programming in Recreational Mathematics collection provides integer programming formulations (using the XPRESS-MP modelling language) for a number of games and puzzles.

In subsequent articles we shall examine in detail the OR/MS content of a number of games and discuss issues related to their integration in OR/MS courseware. To start with, we shall consider the Towers of Hanoi (Sniedovich 2002a) and Counterfeit Coin (Sniedovich 2002b) problems. We derive dynamic programming functional equations for them and show how they can be solved using standard and not so standard dynamic programming methods. Interactive modules are provided for this purpose.

10. Conclusions

There are a number of reasons why games in general and math oriented games in particular should be considered for inclusion in OR/MS lectures. In this discussion we emphasized the very rich educational content that they provide.

As mentioned in the introduction, Mathematicians have long recognized the important role that games can play in education. Given the significant problem-solving orientation of many areas of OR/MS, especially 'optimization', it should not be difficult to establish a rich collection of good 'OR/MS Games'.

What is 'a good OR/MS game'? Well, here again we can seek advice from our colleagues, the mathematicians:

> *In Mathematics, the 'good' problems are those that can be simply stated ... but whose solution are fiendishly difficult. The greater the disparity between the simplicity of the formulation and the complexity of the solution, the 'better' the problem. In this sense, number theory is full of good problems.*
>
> Guedj (2000, p. 243)

This eloquent description of what constitutes a good math problem can guide us in the formulation of selection criteria for good OR/MS problems in general and games in particular. In fact, we have an opportunity to establish OR/MS as an area of applied mathematics that is full of good problems and good games! And to complete the picture, we shall also have to determine which of these problems and games are educationally rich.

Hopefully, this will keep the "Games and Puzzles" section of this journal pretty busy!

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tutORial Project: www.ifors.org/tutorial/

tutORial's Counterfeit Coin problem module: www.ifors.ms.unimelb.edu.au/tutorial/coin/.

tutORial's Towers of Hanoi module: www.ifors.ms.unimelb.edu.au/tutorial/hanoi/.

tutORial's Eight Easy Pieces module: www.ifors.ms.unimelb.edu.au/tutorial/easy/.

tutORial's N-Queen module: www.ifors.ms.unimelb.edu.au/tutorial/queens/.

tutORial's Prince's Pub module: www.ifors.ms.unimelb.edu.au/tutorial/pub/.

tutORial's Gee Park module: www.ifors.ms.unimelb.edu.au/tutorial/gee/.

Yahoo's directory of Math Problems, Puzzles and Games http://dir.yahoo.com/Science/Mathematics/Problems__Puzzles__and_Games/.

